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*Aleksandr Diaczenko**

The gravity model: simulating the interactions in anisotropic space

The gravity model: simulating the interactions in anisotropic space

This paper focuses on a modification of the gravity model regarding the analysis of settlement systems within large regions. The gravity model produces extremely good results for micro-regions; however its application to a large-scale spatial analysis shows divergences between the model and empirical data. The proposed modification of this model that is presented here brings together statistical probabilities of the interactions between different populations and the level of similarities in their material culture. The modified model may be included into network analysis of large-scale spatial data.

KEY WORDS: central places, gravity model, modification, network analysis

INTRODUCTION

The gravity model is being widely used in analytical geography, spatial archaeology and theoretical ecology for the determination of barriers between the catchment areas of different populations, and measuring the interactions between inhabited areas. The model is also applied to the identification of the spread of goods that were produced within the centers, probability of marital migrations etc.

In its classic form, the gravity model describes the interaction of populations within a space of isotropic properties, i.e. a space with equal properties in all directions. This approach seems to be appropriate for micro-regional analysis in spatial archaeology. However, properties of space are often different within large areas. This paper focuses on several research questions. Is there any difference in application of the gravity model to regional versus micro-regional data? Are there any divergences between the empirical and the model data? Is it possible to explain the divergences and, if so, is it possible to modify the gravity model regarding the anisotropic properties of space (within a sense of analytical geography)? Some ideas, expressed below, are exemplified by the materials of the Cucuteni-Tripolye cultural complex (CTCC).

VARIABLES THAT RULE THE INTERACTIONS

First, let us analyze the variables of the gravity model. According to it, the interactions between two settlements are proportional to their populations and reverse to the distance between them, raised to a power. It is written as follows:

$$I = \frac{P_M P_J}{D_{MJ}^a}, \qquad (1)$$

where I is a measure of interactions, and P_M and P_J are the size of populations for settlements M and J. D_{MJ}^a is the distance between two settlements, raised to a power of a (W. J. Reilly 1931).

Studies in analytical geography usually take P_M and P_J for the population values (W. J. Reilly 1931; P. Haggett 1979; A. S. Fotheringham and M. E. O'Kelly 1989; A. Sen and T. E. Smith 1995 et al.), while papers in archaeology also may take these variables for correlates of population estimates (I. Hodder and C. Orton 1976; D. L. Clarke 1977; M. Kolesnikov 2003). This may be exemplified with one of the earliest applications of the gravity model in spatial archaeology, the Cappadocian cities and towns model (W. Tobler and S. Wineburg 1971). W. Tobler and S. Wineburg proposed the following working hypothesis:

1. the mere mention of two towns' names on the same tablet is taken to define a relationship between these towns; and
2. the number of occurrences of a town name is considered to be proportional to the population of that town.

Correlation of the model data with written sources led to the identification of historically known towns among the archaeological sites (W. Tobler and S. Wineburg 1971).

Numerous experiments concerning values of the exponent of a were published. For example, I. Hodder proposed to correlate different types of population mobility with the three values

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of the exponent: $a = 0.5$, $a = 1$, and $a = 2$ (I. Hodder 1974). Two of these values were used in application of the gravity model to the Western Tripolye culture (WTC) settlement systems in the South Bug and Dnieper interfluve: $a = 1$ (A. Diachenko 2010), and $a = 2$ (A. Diachenko and F. Menotti 2012). In the latter case the results obtained show the highest level of correlation between the model data and typo-chronology based on pottery seriation (S. N. Ryzhov 1993; 1999; 2000; 2011; 2012).

It is obvious that interactions decrease as distance between two locations increases. However, empirical data often show that interaction is not just a simple function of distance. Therefore it is important to take into account the nature of the variables, inverse to the distance. Following the gravity model, these variables are the size of populations for two locations. Their product reflects the statistical probability of meetings between each inhabitant of settlement *M* with each inhabitant of settlement *J*. Thus, the variable *I* reflects the ***statistical probability of interactions***. If the values of *k* are stable, no additional factors must be put into the model (fig. 1).

As noted above, application of the gravity model (with $k = 1$) to the analysis of settlements of the CTCC at the micro-regional level led to extremely good results. Stages of the development of settlements were identified within broader cultural phases (A. Diachenko and F. Menotti 2012). More detailed relative chronology of the WTC in the South Bug and Dnieper interfluve recently corroborated these results in the relative chronology of sites belonging to the South Bug local group of the WTC (Tarapata, in press). Application of the gravity model to chronologically different sites led to the identification of probable "genetic links" between the settlements. This means that we have shown the statistical probabilities of new settlement formation by populations that left earlier settlements (A. Diachenko and F. Menotti 2012).

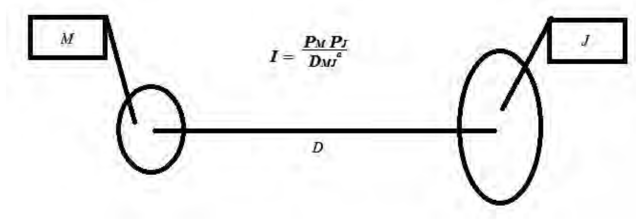

Fig. 1. Interactions in isotropic space. Since no additional factors must be put into the model, interactions between two settlements are proportional to their populations and reverse to the distance between them, raised to a power.

Ryc. 1. Wzajemne oddziaływania w przestrzeni izotropowej. Skoro żadne dodatkowe czynniki nie muszą być wprowadzone do modelu, interakcje między dwoma osadami są proporcjonalne do ich populacji i odwrotnie proporcjonalne do odległości między nimi, podniesione do potęgi.

It should be noted that in the case of the South Bug and Dnieper interfluve we deal with a space of isotropic properties. Would the results of applying the gravity model to larger regions be divergent, if these regions have anisotropic properties?

MODIFICATION OF THE MODEL

The divergences between the empirical and the model data may be exemplified with the materials from the Bodaki settlement belonging to the Shypinetskaya group of the WTC. This site is about 1.5 ha in size, meaning that the population estimates are comparatively low. Meanwhile, according to T.M. Tkachuk, the relative number of imports found at this settlement reaches 17 percent. This value exceeds the number of imports that were found at the large settlements in the South Bug and Dnieper interfluve. Most probably, the attractiveness of Bodaki was caused by the specialization of its population in mining flint and producing long blades (N. N. Skakun et al. 2005). However, this might be only a part of the explanation. W. Christaller noted the existence of hamlets without central functions. Some small agricultural settlements have no economic service functions, but they may serve as social centers or they may have defense purposes (W. Christaller 1966: s. 16, 139–151). Mathematical simulations led to the identification of circumstances influencing the formation of small centers with a population size that does not correlate to the volume of central functions (A. Diachenko, in preparation).

The differing quality of results produced by application of the gravity model to regional and micro-regional scales indicates that modifications are required. One should take into account the anisotropic properties of space. GIS packages make enable the calculation of probable distances between sites in real physical space. However, it is necessary to modify the model, putting within it both the population estimates and the coefficient of similarities in material culture. The first is required by the experience of applications of the gravity model in analytical geography and spatial archaeology, while the second is required by the problem that was mentioned above. Both factors could be considered if we take *k* in equation 1 as not a stable value, but as a variable. It may be calculated as the sum of 1 and the variable *q*, which reflects the level of similarities in material culture. This is written as follows:

$$k = 1 + q, \tag{2}$$

where 0 « *q* « 1.

Numerous ways of calculating the similarity coefficient exist as of today. One example is presented in Equation 3.

$$q = \left(\sum_{i=1}^{n} \sqrt{p_i g_i} \right) \tag{3}$$

where *p* and *g* are the relative frequencies of the presence of a feature in material complexes of two sites (A. I. Martynov and Ya. A. Sher 2002).

Thus, the modified gravity model may be expressed as follows:

$$I = (1 + q)\left(\frac{P_I P_J}{D_{MJ}^2} \right) \tag{4}$$

or

$$I = \left(1 + \sum_{i=1}^{n} \sqrt{p_i g_i} \right)\left(\frac{P_I P_J}{D_{MJ}^2} \right) \tag{5}$$

Now, let us consider two hypothetical extremum cases, when the material culture of the sites is absolutely different or it is absolutely identical (in other words, $q = 0$ and $q = 1$). The variable k is equal to 1 and 2, respectively.

Theoretically speaking, two settlements may not adopt a culture of each other. In the first scenario $q = 0$, $k = 1$, and the statistical probability of the interactions between two settlements is proportional to their populations, inverse to the distance between them raised to a power.

As a second extremum, material culture of two settlements could be absolutely identical. For example, the goods may be produced in a center and used in both the center and a local village. In this case $q = 1$, $k = 2$, and the statistical probability of interactions, known empirically, is twice as high as the model result. This modification makes it possible to close the gap between statistical probabilities and real data. Mathematical derivatives of this modification regarding the application of graph theory and network analysis to large-scale spatial data are discussed below.

CONCLUSION AND DISCUSSION

The gravity model shows extremely good results when applied to sites on a micro-regional level. Meanwhile, the model data differs much from empirical data in the case of large regions. Most probably, these divergences are explained with the anisotropic properties of space according to the conventions of analytical geography. Therefore one possible variants of gravity model modification was proposed. This modification brings together the traditional statistical probabilities of interactions between prehistoric settlements with corrections made in reference to empirical data. Application of the model to measuring graphs has clear mathematical derivatives.

If the settlements are highly clustered in space and the settlement hierarchy in micro-regions is well-developed, then the clusters of sites could be viewed as small-world networks. Elements within the clusters are highly interconnected, but only a few elements are connected to the elements in other clusters (fig. 2). Statistical properties of small-world networks are described with power-law functions (A. Albert and A. L. Barabasi 2002; R. A. Bentley 2003; R. A. Bentley and S. J. Shennan 2003; 2005; E. Ravasz and A. L. Barabasi 2003; V. Colliza et al 2004 et al.). This property is very important for mathematical simulations in archaeology, because it could help to fill the gaps in our (almost always) fragmentary data. Moreover, combining network analysis the Central Place Theory into a single research procedure could enable the reconstruction of both horizontal and vertical links between settlements (O. Nakoinz 2012).

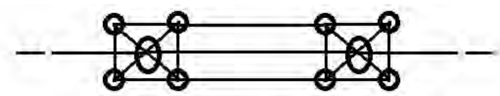

Fig. 2. Small-world network. Number of edges (l) between the nodes follows the power-law function: $C(l) \sim l^{-b}$.

Ryc. 2. Układ małego świata. Liczba krawędzi (l) między wierzchołkami wynika z funkcji prawa potęgowego: $C(l) \sim l^{-b}$.

However, one should remember the fact that settlement systems other than hierarchical types could also exist within large scale regions. In this case a set of networks has different statistical properties and requires additional mathematical analysis (R. M. May 2006).

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Aleksandr Diachenko

Model grawitacyjny: symulacja wzajemnego oddziaływania w przestrzeni anizotropicznej

Streszczenie

Artykuł ten skupia się na modyfikacji modelu grawitacyjnego w odniesieniu do analizy systemów osadniczych w ramach dużych regionów. Model grawitacyjny dostarcza bardzo dobrych wyników dla mikroregionów, jednak jego stosowanie w stosunku do analizy przestrzennej, na dużą skalę, pokazuje rozbieżności pomiędzy modelem a danymi empirycznymi. Proponowana zmiana tego modelu, który został tu przedstawiony, łączy dane statystyczne w odniesieniu do prawdopodobieństwa interakcji pomiędzy różnymi populacjami oraz poziomu podobieństwa ich kultury materialnej. Zmodyfikowany model może być włączony do analizy układu przestrzennego na dużą skalę.